Steel and Concrete Design

FE Review Session

By Jake Stevens

Problem 1: Load Combinations

A steel W-section is uniformly loaded to produce bending about its strong axis. The beam supports a uniformly distributed service dead load of 2.5 kips/ft and a service live load of 1.8 kips/ft on a 36 ft simple span. What is most nearly the design load?

Problem 1 Solution:

Load Combinations using Strength Design (LRFD)

Basic combinations

(L_r or S or R) = largest of L_r, S, R

(L or $0.5W$) = larger of L, $0.5W$

Nominal loads used in the following combinations

$1.4D$

$1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$

$1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$

$1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$

$1.2D + 1.0E + L + 0.2S$

$0.9D + 1.0W$

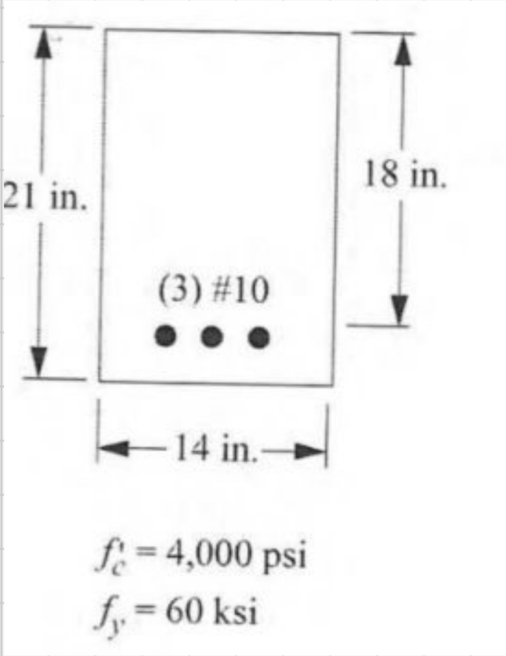
$0.9D + 1.0E$

Combo 1:
U = 1.4D = 1.4(2.5 kip/ft)= 3.5 kip/ft

Combo 2:
U = 1.2D+1.6L
U = 1.2(2.5 kip/ft)+1.6(1.8 kip/ft)
U = 5.9 kip/ft

Choose the larger value: **5.9 ft/ft**

Problem 2: Reinforced Concrete Beam Flexure



The Flexural design strength (kips*ft) of the reinforced concrete beam section shown is most nearly:

A) 267
B) 297
C) 319
D) 354

Problem 2: Equations and Terms

Singly-Reinforced Beams

$$\varepsilon_t = \frac{0.003(d_t - c)}{c} = \frac{0.003(\beta_1 d_t - a)}{a}$$

$$a = \frac{A_s\, f_y}{0.85\, f_c'\, b}$$

$$M_n = 0.85\, f_c'\, a\, b\left(d - \frac{a}{2}\right) = A_s f_y\left(d - \frac{a}{2}\right)$$

a = depth of equivalent rectangular stress block (in.)

b = width of compression face of member (in.)

A_{st} = total area of longitudinal reinforcement (in^2)

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement (in.)

f'_c = compressive strength of concrete (psi)

f_y = yield strength of steel reinforcement (psi)

M_n = nominal flexural strength at section

β_1 = ratio of depth of rectangular stress block a to depth to neutral axis c

Problem 2 Solution

ASTM STANDARD REINFORCEMENT BARS

BAR SIZE	DIAMETER, IN.	AREA, IN^2	WEIGHT, LB/FT
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
#6	0.750	0.44	1.502
#7	0.875	0.60	2.044
#8	1.000	0.79	2.670
#9	1.128	1.00	3.400
#10	1.270	1.27	4.303
#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.60

A_{st}= 3#10 bars * 1.27 in^2 = 3.81in^2

f'_c= 4 ksi

f_y= 60 ksi

b= 14 in

d= 18 in

a= [3.81 in^2 *60 ksi]/[0.85*4 ksi* 14 in] = 4.8 in

M_n= 3.81 in^2 * 60 ksi (18in-4.8/2) = 3564 kip*in

M_n= 297 kip*ft

Problem 2 Solution

$f'_c\,(psi)$	β_1	
$2500 \le f'_c \le 4000$	0.85	(a)

Resistance Factors, ϕ

Tension-controlled sections ($\varepsilon_t \ge 0.005$):	$\phi = 0.9$
Compression-controlled sections ($\varepsilon_t \le 0.002$):	
Members with tied reinforcement	$\phi = 0.65$
Transition sections ($0.002 < \varepsilon_t < 0.005$):	
Members with tied reinforcement	$\phi = 0.48 + 83\varepsilon_t$
Shear and torsion	$\phi = 0.75$
Bearing on concrete	$\phi = 0.65$

β = 0.85 (from chart)

ε = [0.003(0.85*18 in- 4.8 in)]/4.8 = 0.0066

ϕ = 0.9 (from chart)

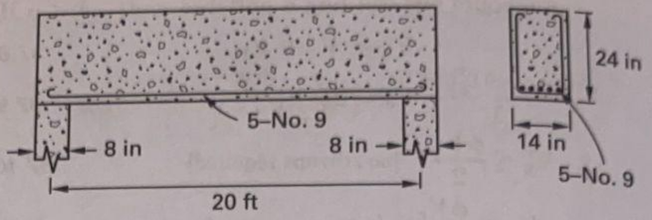
ϕM_n = 0.9*297 kip*ft

ϕM_n = 267 kip*ft

Problem 3: Reinforce Concrete beam Shear Design

Example

The simply supported beam shown spans 20 ft and, in addition to its own weight, carries a uniformly distributed service dead load of 1.75 kips/ft and a uniformly distributed live load of 3.0 kips/ft. Five no. 9 bars running the full length of the beam are used as flexural reinforcement. $f'_c = 4000$ lbf/in^2, and $f_y = 60{,}000$ lbf/in^2. Number 3 stirrups are used. The concrete cover on longitudinal steel is 2.5 in.



What is most nearly the theoretical spacing of the stirrups at a point where the required shear strength is 60.20 kips?

(A) 2.4 in

(B) 2.9 in

(C) 6.7 in

(D) 13 in

Problem 3: Equations and Terms

Beams—Shear

$$\phi V_n \geq V_u$$

Nominal shear strength:

$$V_n = V_c + V_s$$

$$V_c = 2\lambda\sqrt{f_c'}\,b_w d$$

s = center to center spacing of longitudinal shear or torsional reinforcement (in.)

V_c = nominal shear strength provided by concrete (lb)

V_n = nominal shear strength at section (lb)

ϕV_n = design shear strength at section (lb)

V_s = nominal shear strength provided by reinforcement (lb)

V_u = factored shear force at section (lb)

λ = 1.0 for normal weight concrete (NWC)

λ = 0.75 for lightweight concrete

$V_s = \dfrac{A_v f_y d}{s}$ (may not exceed $8\, b_w\, d\sqrt{f_c'}$)

b = width of compression face of member (in.)

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement (in.)

Problem 3: Solution

V_u= 60.2 kips
f'_c= 4000 psi
f_y= 60 ksi
b= 14 in
ϕ= 0.75 (shear)

d = 24 in- 2.5 in = 21.5 in
A_{vs}= 2*Area of#3 bar = 2*0.11 in^2 = 0.22in^2
V_n = 60.2 kips/0.75 = 80.23 kips
V_c= 2*(1)*√(4000 psi)* 14 in* 21.5 in = 38.07 kips
V_s= V_n-V_c = 80.23kip - 38.07 kip = 42.2 kips

s= [A_v* f_y* d]/V_s
s= [0.22in^2 *60 ksi* 21.5 in]/ 42.2 kips
s= 6.73 in

Problem 4: Reinforced Concrete Column

5. An 18 in square tied column is reinforced with 12 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in^2. The column, which is braced against sidesway, has an unsupported height of 9 ft and supports axial load only without end moments. What is most nearly the design axial load capacity?

(A) 930 kips

(B) 970 kips

(C) 1800 kips

(D) 1900 kips

Problem 4 Equations and Terms:

Design Column Strength, Tied Columns

$$\phi P_n = 0.80\phi[0.85\, f_c'(A_g - A_{st}) + A_{st}\, f_y]$$

A_g = gross area of concrete section (in^2)
A_{st} = total area of longitudinal reinforcement (in^2)
f'_c = compressive strength of concrete (psi)
f_y = yield strength of steel reinforcement (psi)

Compression-controlled sections ($\varepsilon_t \le 0.002$):
Members with tied reinforcement $\phi = 0.65$

ASTM STANDARD REINFORCEMENT BARS

BAR SIZE	DIAMETER, IN.	AREA, IN^2	WEIGHT, LB/FT
#3	0.375	0.11	0.376
#4	0.500	0.20	0.668
#5	0.625	0.31	1.043
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#11	1.410	1.56	5.313
#14	1.693	2.25	7.650
#18	2.257	4.00	13.60

Problem 4 Solution:

A_{st}= 12#9 bars * 1 in^2 = 12 in^2

A_g= 18 in* 18in = 324 in^2

f'_c= 4 ksi

f_y= 60 ksi

ϕP_n= 0.8(0.65)[0.85(4 ksi)(324 in^2 -12 in^2)+ 12 in^2(60 ksi)]

ϕP_n= 930 kips

Problem 5: Steel Tension Members

2. A steel tension member is 5 in long and $\frac{1}{2}$ in thick. There are two holes in the bar. The holes are in parallel and are made for a $\frac{1}{4}$-in bolt each. The net area is most nearly

(A) 2.0 in^2

(B) 2.2 in^2

(C) 2.5 in^2

(D) 2.7 in^2

Problem 5: Equations and Terms

Flat Bars or Angles, Bolted or Welded

Definitions

Bolt diameter: d_b

Nominal hole diameter: $d_h = d_b + \frac{1}{16}"$

Gross width of member: b_g

Member thickness: t

Connection eccentricity: $\overline{x}$

Gross area: $A_g = b_g\, t$ (use tabulated areas for angles)

Net area (parallel holes): $A_n = \left[b_g - \Sigma\left(d_b + \frac{1}{16}" \right) \right] t$

Net area (staggered holes):

$$A_n = \left[b_g - \Sigma\left(d_b + \frac{1}{16}" \right) + \Sigma \frac{s^2}{4g} \right] t$$

s = longitudinal spacing of consecutive holes

g = transverse spacing between lines of holes

Limit States and Available Strengths

Yielding:

$\phi_y = 0.90$
$P_n = F_y A_g$
$\Omega = 1.67$

Rupture:

$\phi_f = 0.75$
$P_n = F_u A_e$
$\Omega = 2.00$

Block shear:

$\phi = 0.75$
$\Omega = 2.00$
$U_{bs} = 1.0$ (flat bars and angles)
A_{gv} = gross area for shear
A_{nv} = net area for shear
A_{nt} = net area for tension

$$R_n = \text{Smaller} \begin{cases} F_u[0.6A_{nv} + U_{bs}A_{nt}] \\ [0.6F_y A_{gv} + U_{bs} F_u A_{nt}] \end{cases}$$

Problem 5 Solution:

b_g= 5 in
t= ½ in
d_b= 1/4 in

A_n= (5in - 2 bolts (0.25in + 0.0625))* ½ in = **2.2 in^2**

Problem 6: Steel Members in Bending

Example

A W21 × 48 beam of A992 steel has a compression flange braced at 6 ft intervals. Assume the beam is compact. What is most nearly the available plastic moment capacity of the beam?

(A) 360 ft-kips

(B) 400 ft-kips

(C) 410 ft-kips

(D) 420 ft-kips

Problem 6 Solution:

Yielding

$$M_n = M_p = F_y Z_x$$

where

F_y = specified minimum yield stress

Z_x = plastic section modulus about the x-axis

Shape	Area	Depth	Web	Flange		Axis X-X				Axis Y-Y	
	A	d	t_w	b_f	t_f	I	S	r	Z	I	r
	In.^{2}	In.	In.	In.	In.	In.^{4}	In.^{3}	In.	In.^{3}	In.^{4}	In.
W24X68	20.1	23.7	0.415	8.97	0.585	1830	154	9.55	177	70.4	1.87
W24X62	18.2	23.7	0.430	7.04	0.590	1550	131	9.23	153	34.5	1.38
W24X55	16.3	23.6	0.395	7.01	0.505	1350	114	9.11	134	29.1	1.34
W21X73	21.5	21.2	0.455	8.30	0.740	1600	151	8.64	172	70.6	1.81
W21X68	20.0	21.1	0.430	8.27	0.685	1480	140	8.60	160	64.7	1.80
W21X62	18.3	21.0	0.400	8.24	0.615	1330	127	8.54	144	57.5	1.77
W21X55	16.2	20.8	0.375	8.22	0.522	1140	110	8.40	126	48.4	1.73
W21X57	16.7	21.1	0.405	6.56	0.650	1170	111	8.36	129	30.6	1.35
W21X50	14.7	20.8	0.380	6.53	0.535	984	94.5	8.18	110	24.9	1.30
W21X48	14.1	20.6	0.350	8.14	0.430	959	93.0	8.24	107	38.7	1.66
W21X44	13.0	20.7	0.350	6.50	0.450	843	81.6	8.06	95.4	20.7	1.26

f_y=50 ksi

Z_x= 107 in^3

ϕ = 0.9

ϕM_p= 0.9*50 ksi*107 in^3 = **401 kip*ft**

Problem 7: Structural Steel Lateral Torsional Buckling

Example

A W21 × 68 beam is 32 ft long. The beam is made of A992 steel, which has a yield strength of 50 ksi. The compression flange is braced at the ends and at intervals of 8 ft. C_b is 1.0, and ϕM_p is 600 ft-kips. L_p is 6.36 ft, and L_r is 18.7 ft. The service loading results in a 256 ft-kips moment due to uniform loading (live), an 80 ft-kips moment due to point loads (live), and an 8.704 ft-kips moment due to the beam's self-weight. What is most nearly the available moment strength, and is the beam adequate?

(A) 550 ft-kips; beam is not adequate

(B) 550 ft-kips; beam is adequate

(C) 570 ft-kips; beam is not adequate

(D) 570 ft-kips; beam is adequate

Problem 7 Equations and Terms

Lateral-Torsional Buckling

Based on bracing where L_b is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits L_p and L_r:

When $L_b \le L_p$, the limit state of lateral-torsional buckling does not apply.

When $L_p < L_b \le L_r$

$$M_n = C_b\left[M_p - (M_p - 0.7\,F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$

where

$$C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_{\text{A}} + 4M_{\text{B}} + 3M_{\text{C}}}$$

M_{max} = absolute value of maximum moment in the unbraced segment
M_A = absolute value of maximum moment at quarter point of the unbraced segment
M_B = absolute value of maximum moment at centerline of the unbraced segment
M_C = absolute value of maximum moment at three-quarter of the unbraced segment

Shape	Area	Depth	Web	Flange		Axis X-X				Axis Y-Y	
	A	d	t_w	b_f	t_f	I	S	r	Z	I	r
	$In.^2$	In.	In.	In.	In.	$In.^4$	$In.^3$	In.	$In.^3$	$In.^4$	In.
W24X68	20.1	23.7	0.415	8.97	0.585	1830	154	9.55	177	70.4	1.87
W24X62	18.2	23.7	0.430	7.04	0.590	1550	131	9.23	153	34.5	1.38
W24X55	16.3	23.6	0.395	7.01	0.505	1350	114	9.11	134	29.1	1.34
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W21X68	20.0	21.1	0.430	8.27	0.685	1480	140	8.60	160	64.7	1.80

Yielding

$$M_n = M_p = F_y Z_x$$

where

F_y = specified minimum yield stress
Z_x = plastic section modulus about the x-axis

Problem 7 Solution

f_y=50 ksi
s_x= 140 in^3
Z_x= 160 in^3
M_p= 50 ksi*160 in^3 = 667 kip*ft
M_n = 1* [667-(667- 0.7(50 ksi)(140in^3)/12)[(8ft-6.36ft)/(18.7ft-6.36ft)]
M_n = 632.6 kip*ft
ϕM_n = 0.9*632.6 kip*ft = **569.4 kip*ft** (Design Moment Capacity)

M_u= 1.2(8.704 kip*ft)+1.6(256+80 kip*ft) = 548.1 kip*ft

Since the design moment is greater than the ultimate moment the beam is adequate to support the load

Problem 8: Steel Column Buckling

Structural

4. A steel column is built-in at one end and free to translate and rotate at the other end. The column uses a 12 ft long W12 × 45 beam. If the yield strength of the steel is 50 $kips/in^2$, the critical stress in the column is most nearly

(A) 7.3 $kips/in^2$

(B) 9.0 $kips/in^2$

(C) 9.7 $kips/in^2$

(D) 10 $kips/in^2$

Problem 8: Equations and terms

Columns

The *design compressive strength* $\phi_c P_n$ is determined with $\phi_c = 0.90$ and $\Omega = 1.67$ for flexural buckling of members without slender elements and is determined as follows:

$$P_n = F_{cr} A_g$$

where the critical stress F_{cr} is determined as follows:

(a) When $\frac{Lc}{r} \leq 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = \left[0.658^{F_y/F_e}\right] F_y$

(b) When $\frac{Lc}{r} > 4.71\sqrt{\frac{E}{F_y}}$, $F_{cr} = 0.877\, F_e$

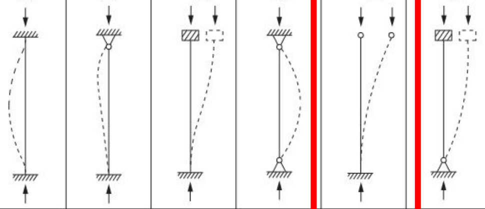
where

$Lc = KL =$ effective length of member (in.)

$F_e =$ elastic buckling stress $= \pi^2 E/(KL/r)^2$

Problem 8: Equations and terms

TABLE C-A-7.1
AISC APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, *K*

BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.	(a)	(b)	(c)	(d)	(e)	(f)
						
THEORETICAL *K* VALUE	0.5	0.7	1.0	1.0	2.0	2.0
AISC-RECOMMENDED DESIGN VALUE WHEN IDEAL CONDITIONS ARE APPROXIMATED	0.65	0.80	1.2	1.0	2.10	2.0
END CONDITION CODE	ROTATION FIXED AND TRANSLATION FIXED; ROTATION FREE AND TRANSLATION FIXED; ROTATION FIXED AND TRANSLATION FREE; ROTATION FREE AND TRANSLATION FREE					

k= 2.1

W12X79	23.2	12.4	0.470	12.1	0.735	662	107	5.34	119	216	3.05
W12X72	21.1	12.3	0.430	12.0	0.670	597	97.4	5.31	108	195	3.04
W12X65	19.1	12.1	0.390	12.0	0.605	533	87.9	5.28	96.8	174	3.02
W12X58	17.0	12.2	0.360	10.0	0.640	475	78.0	5.28	86.4	107	2.51
W12X53	15.6	12.1	0.345	9.99	0.575	425	70.6	5.23	77.9	95.8	2.48
W12X50	14.6	12.2	0.370	8.08	0.640	391	64.2	5.18	71.9	56.3	1.96
W12X45	13.1	12.1	0.335	8.05	0.575	348	57.7	5.15	64.2	50.0	1.95
W12X40	11.7	11.9	0.295	8.01	0.515	307	51.5	5.13	57.0	44.1	1.94

r_x= 5.15 in
r_y=1.95 in
Choose the smaller value because it will have a larger slenderness ratio

Problem 8: Solution

k= 2.1 in

r= 1.95 in

L = 12 ft = 144 in

Slenderness ratio = (2.1*144)/1.95 = 155.1

F_y = 50 ksi

4.71√(29,000 ksi/50 ksi) = 113.4

Since the slenderness ratio is greater than 113.4, use the second equation

$F_e = (29{,}000 \cdot \pi^2)/(155.1)^2$ = 11.9 ksi

F_{cr} = 0.877(11.9 ksi) = **10.44 ksi**